Marc's original

1_NOTES.PDF

Brushless DC Motor Model

1

(a) A model of a three-phase synchronous motor is

$$
\begin{aligned}
L\frac{di_A}{dt} - M\frac{di_B}{dt} - M\frac{di_C}{dt} &= v_A - Ri_A + K\omega\sin(\theta) \\
-M\frac{di_A}{dt} + L\frac{di_B}{dt} - M\frac{di_C}{dt} &= v_B - Ri_B + K\omega\sin(\theta - 120^o) \\
-M\frac{di_A}{dt} - M\frac{di_B}{dt} + L\frac{di_C}{dt} &= v_C - Ri_C + K\omega\sin(\theta - 240^o) \\
J\frac{d\omega}{dt} &= -Ki_A\sin(\theta) - Ki_B\sin(\theta - 120^o) - Ki_C\sin(\theta - 240^o) - \tau_L
\end{aligned}
$$

If the phases were perfectly coupled, one would have $M = \frac{1}{2}L$.

(b) The three-phase to two-phase transformation

$$
\begin{aligned}
i_{A'} &= i_A - \frac{1}{2}\, i_B - \frac{1}{2}\, i_C \\
i_{B'} &= \frac{\sqrt{3}}{2} i_B - \frac{\sqrt{3}}{2} i_C
\end{aligned}
$$

transforms the original model into a two-phase model

$$
\begin{aligned}
(L+M)\frac{di_{A'}}{dt} &= v_{A'} - Ri_{A'} + \frac{3}{2}K\omega\sin(\theta) \\
(L+M)\frac{di_{B'}}{dt} &= v_{B'} - Ri_{B'} - \frac{3}{2}K\omega\cos(\theta) \\
J\frac{d\omega}{dt} &= -Ki_{A'}\sin(\theta) + Ki_{B'}\cos(\theta) - \tau_L
\end{aligned}
$$

The equivalent inductance $L_{eq} = L + M$ is approximately $\frac{3}{2}L$.

(c) The three-phase DQ transformation

$$
\begin{aligned}
i_d &= \cos(\theta)\, i_A + \cos(\theta - 120^o)\, i_B + \cos(\theta - 240^o)\, i_C \\
i_q &= -\sin(\theta)\, i_A - \sin(\theta - 120^o)\, i_B - \sin(\theta - 240^o) i_C \\
i_h &= i_A + i_B + i_C
\end{aligned}
$$

is the cascade of the three-phase to two-phase transformation, together with $i_h = i_A + i_B + i_C$, and a 2-phase DQ transformation from $i_{A'}$ and $i_{B'}$ to i_d and i_q, leaving i_h unchanged. The inverse of the DQ transformation is

$$
\begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \frac{2}{3} \begin{pmatrix} \cos(\theta) & -\sin(\theta) & \frac{1}{2} \\ \cos(\theta - 120^o) & -\sin(\theta - 120^o) & \frac{1}{2} \\ \cos(\theta - 240^o) & -\sin(\theta - 240^o) & \frac{1}{2} \end{pmatrix} \begin{pmatrix} i_d \\ i_q \\ i_h \end{pmatrix}
$$

Marc's Original

2

and one can show that

$$v_A i_A + v_B i_B + v_C i_C = \frac{2}{3}\left(v_d i_d + v_q i_q\right) + \frac{1}{3} v_h i_h$$

The model in the DQ coordinates is

$$\begin{aligned} L_{eq}\frac{di_d}{dt} &= v_d - Ri_d + \omega L_{eq} i_q \\ L_{eq}\frac{di_q}{dt} &= v_q - Ri_q - \omega L_{eq} i_d - \frac{3}{2}K\omega \\ J\frac{d\omega}{dt} &= Ki_q - \tau_L \end{aligned}$$

with an independent equation for the homopolar current

$$L\frac{di_h}{dt} = v_h - Ri_h$$

The power converted is $K\omega i_q$. For a motor with N_P pole pairs, replace θ by $N_P\theta$ in the original model, and ωL_{eq} by $N_R\omega L_{eq}$ in the DQ model.

Brushless DC Motor Model

3

(a) A model of a three-phase synchronous motor is

$$\begin{aligned}
L\frac{di_{S1}}{dt} - M\frac{di_{S2}}{dt} - M\frac{di_{S3}}{dt} &= v_{S1} - Ri_{S1} + K\omega\sin(\theta) \\
-M\frac{di_{S1}}{dt} + L\frac{di_{S2}}{dt} - M\frac{di_{S3}}{dt} &= v_{S2} - Ri_{S2} + K\omega\sin(\theta - \frac{2\pi}{3}) \\
-M\frac{di_{S1}}{dt} - M\frac{di_{S2}}{dt} + L\frac{di_{S3}}{dt} &= v_{S3} - Ri_{S3} + K\omega\sin(\theta - \frac{4\pi}{3}) \\
J\frac{d\omega}{dt} &= -Ki_{S1}\sin(\theta) - Ki_{S2}\sin(\theta - \frac{2\pi}{3}) - Ki_{S3}\sin(\theta - \frac{4\pi}{3}) - \tau_L
\end{aligned}$$

If the phases were perfectly coupled, one would have $M = \frac{1}{2}L$.

(b) The three-phase to two-phase transformation

$$\begin{bmatrix} i_a \\ i_b \\ i_0 \end{bmatrix} \triangleq \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} i_{S1} \\ i_{S2} \\ i_{S3} \end{bmatrix}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_0 \end{bmatrix} \triangleq \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} v_{S1} \\ v_{S2} \\ v_{S3} \end{bmatrix}$$

transforms the original model into the equivalent model

$$\begin{aligned}
(L+M)\frac{di_a}{dt} &= v_a - Ri_a + \sqrt{\frac{3}{2}}K\omega\sin(\theta) \\
(L+M)\frac{di_b}{dt} &= v_b - Ri_b - \sqrt{\frac{3}{2}}K\omega\cos(\theta) \\
(L-2M)\frac{di_0}{dt} &= \frac{1}{\sqrt{3}}v_0 - \frac{1}{\sqrt{3}}Ri_0 \\
J\frac{d\omega}{dt} &= -\sqrt{\frac{3}{2}}Ki_a\sin(\theta) + \sqrt{\frac{3}{2}}Ki_b\cos(\theta) - \tau_L
\end{aligned}$$

For a balanced three-phase system in which $v_0 = v_{S1} + v_{S2} + v_{S3} = 0$, $i_0 = i_{S1} + i_{S2} + i_{S3} = 0$, one obtains the *two-phase equivalent* model given by

$$\begin{aligned}
L_{eq}\frac{di_a}{dt} &= v_a - Ri_a + K_{eq}\omega\sin(\theta) \\
L_{eq}\frac{di_b}{dt} &= v_b - Ri_b - K_{eq}\omega\cos(\theta) \\
J\frac{d\omega}{dt} &= -K_{eq}i_a\sin(\theta) + K_{eq}i_b\cos(\theta) - \tau_L
\end{aligned}$$

where the equivalent inductance $L_{eq} = L + M$ is approximately $\frac{3}{2}L$ and $K_{eq} = \sqrt{\frac{3}{2}}K$ is the equivalent torque/back-emf constant.

(c) The DQ transformation

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$

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One can show that

$$v_{S1}i_{S1} + v_{S2}i_{S2} + v_{S3}i_{S3} = v_d i_d + v_q i_q$$

The model in the DQ coordinates is

$$\begin{aligned} L_{eq}\frac{di_d}{dt} &= v_d - Ri_d + \omega L_{eq} i_q \\ L_{eq}\frac{di_q}{dt} &= v_q - Ri_q - \omega L_{eq} i_d - K_{eq}\omega \\ J\frac{d\omega}{dt} &= K_{eq} i_q - \tau_L \end{aligned}$$

The electrical power converted to mechanical power is $K_{eq}\omega i_q$. For a motor with n_P pole pairs, replace θ by $n_P\theta$ in the original model, and ωL_{eq} by $n_p \omega L_{eq}$ in the DQ model.

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$$\frac{di_1}{dt} = \left[v_1 - R i_1 + K_m \omega \sin(N_r\theta)\right]/L$$

$$\frac{di_2}{dt} = \left[v_2 - R i_2 + K_m \omega \sin(N_r\theta - \gamma)\right]/L$$

$$\frac{di_3}{dt} = \left[v_3 - R i_3 + K_m \omega \sin(N_r\theta - 2\gamma)\right]/L$$

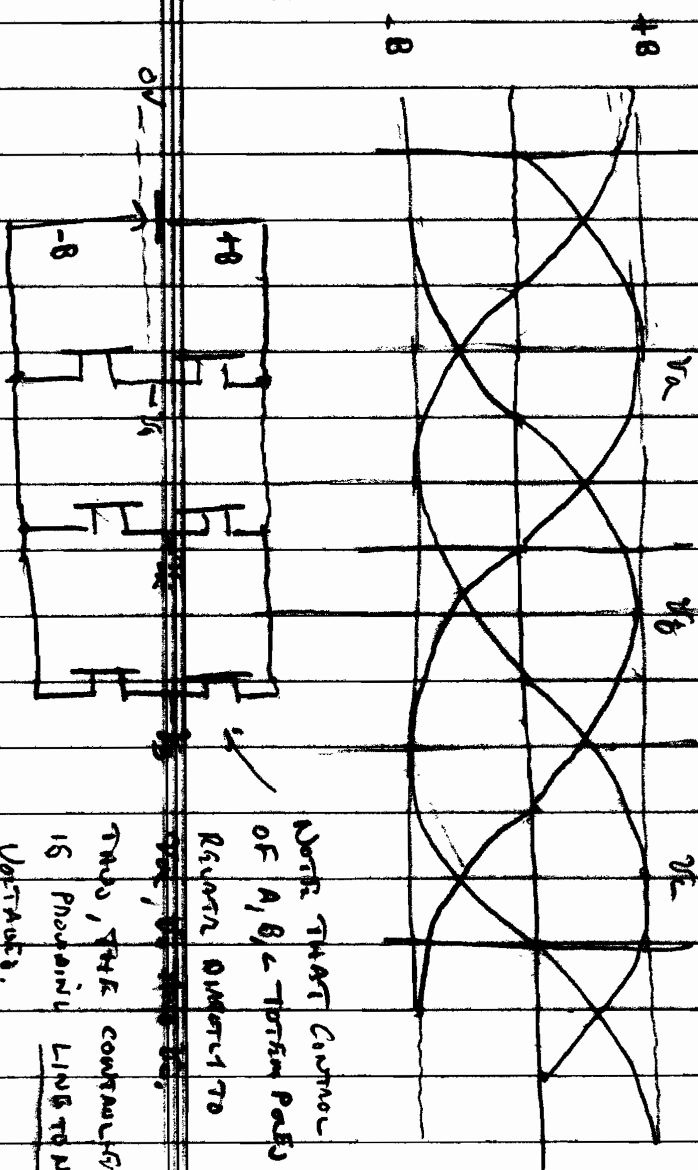
MISSING $-B\omega$

$$\frac{d\omega}{dt} = \left[-K_m i_1 \sin(N_r\theta) - K_m i_2 \sin(N_r\theta - \gamma) - K_m i_3 \sin(N_r\theta - 2\gamma)\right]/J$$

$$\frac{d\theta}{dt} = \omega$$

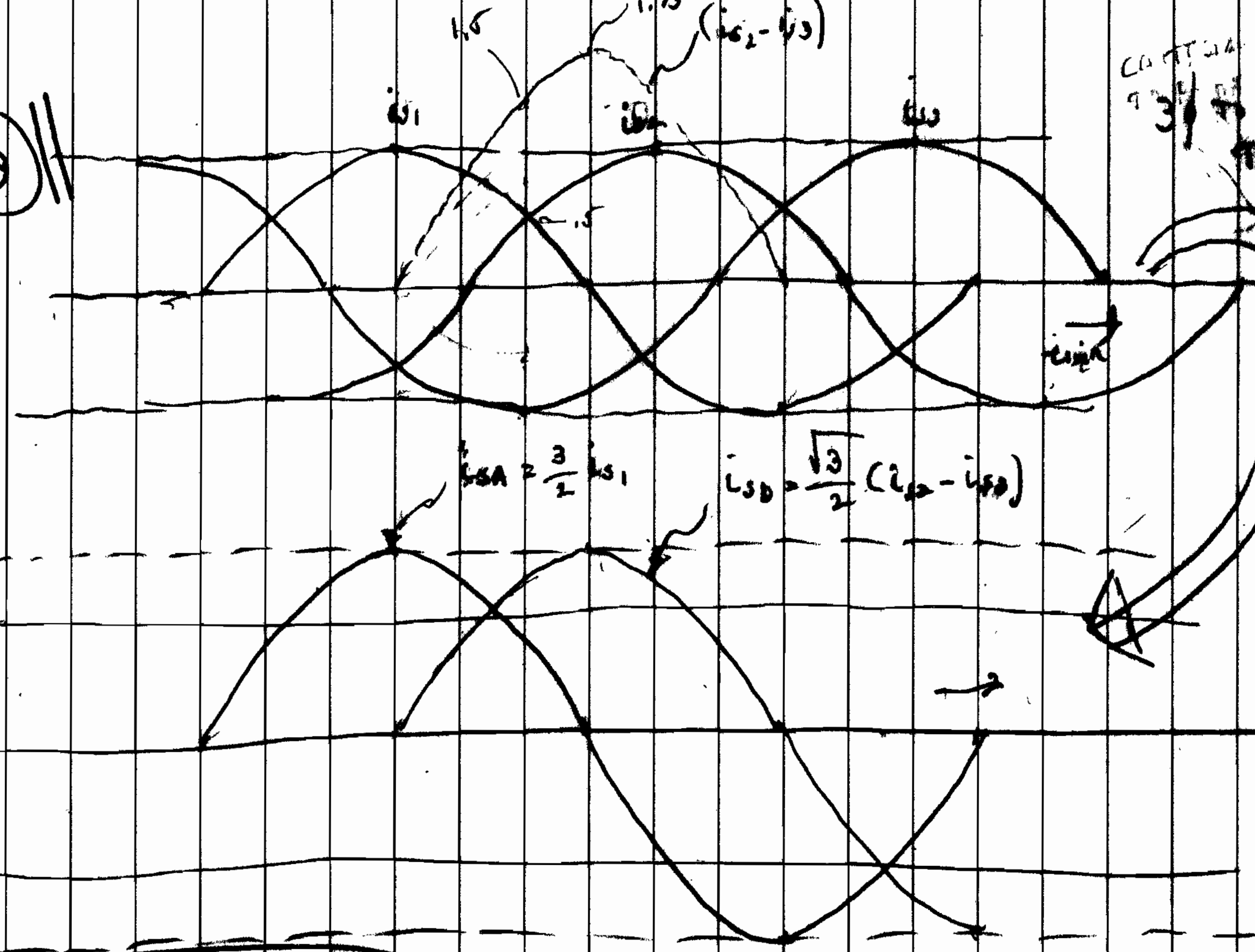
WHERE: $\gamma = \frac{2\pi}{3}$

(NOTE: $i_1^2 + i_2^2 + i_3^2 = 1.5$ FOR 1 pu SINE WAVE; THUS THE TORQUE IS 50% GREATER THAN A 2 PHASE MOTOR RUNNING AT THE SAME CURRENT)



NOTE THAT CONTROL OF A, B, C TOTEM POLES RELATIVE ONE TO ANOTHER, THUS, THE CONTROLLER IS PROVIDING LINE TO NEUTRAL VOLTAGES.

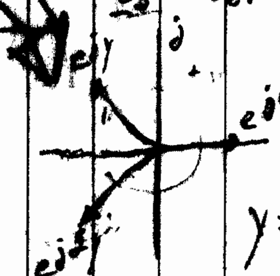
(6)



CONTINUE ... MATRIX $3\phi \to 2\phi$ TRANSFORMATION

$e^{j\theta} = \cos\theta + j\sin\theta$

$\underline{i}_s = i_{s1} + i_{s2} e^{j\gamma} + i_{s3} e^{j2\gamma}$



$\gamma = \frac{2\pi}{3} = 120°$

$e^{j\frac{2\pi}{3}} = \cos(120) + j\sin(120)$
$= -.5 + j.866$

$e^{j\frac{4\pi}{3}} = \cos(240) + j\sin(240)$
$= -.5 - j.866$

$= -j$

$\underline{i}_s = i_{s1} - .5\, i_{s2} + j.866\, i_{s2}$
$- .5\, i_{s3} - j.866\, i_{s3} = i_{s\alpha} + j i_{s\beta}$

ASSUMING i_{s1}, i_{s2}, i_{s3} ARE SYMMETRICAL

$i_{s\alpha} = \frac{3}{2} i_{s1}$ — THIS MATCHES WITH PAGE (1) NOTES.

$i_{s\beta} = \frac{\sqrt{3}}{2}(i_{s2} - i_{s3})$

FROM PAGE 217 OF LEONHARD "CONTROL OF ELECTRIC DRIVES"

IN MATRIX FORM

$$\begin{vmatrix} i_{s\alpha} \\ i_{s\beta} \end{vmatrix} = \begin{vmatrix} 1.5 & 0 & 0 \\ 0 & .866 & -.866 \end{vmatrix} \begin{vmatrix} i_{s1} \\ i_{s2} \\ i_{s3} \end{vmatrix}$$

AND

$$\begin{vmatrix} V_{s\alpha} \\ V_{s\beta} \end{vmatrix} = \begin{vmatrix} 1.5 & 0 & 0 \\ 0 & .866 & -.866 \end{vmatrix} \begin{vmatrix} V_{s1} \\ V_{s2} \\ V_{s3} \end{vmatrix}$$

AND

$$\begin{vmatrix} \frac{di_{s\alpha}}{dt} \\ \frac{di_{s\beta}}{dt} \end{vmatrix} = \begin{vmatrix} 1.5 & 0 & 0 \\ 0 & .866 & -.866 \end{vmatrix} \begin{vmatrix} \frac{di_{s1}}{dt} \\ \frac{di_{s2}}{dt} \\ \frac{di_{s3}}{dt} \end{vmatrix}$$

7

$$v_{sa} = 1.5 R i_1 + 1.5 \frac{di_1}{dt} L - 1.5 K_m \omega \sin(N_r\theta)$$

$$v_{sb} = .866 R i_2 + .866 R i_3 + .866 \frac{di_2}{dt} L - .866 \frac{di_3}{dt} L - .866 K_m \sin(N_r\theta - \gamma) + .866 K_m \sin(N_r\theta - 2\gamma)$$

$$i_1 = \frac{2}{3} i_{sa}$$

$$i_2 - i_3 = \frac{i_{sb}}{.866}$$

$$= 1.5 \cos(N_r\theta)$$

$\frac{di_a}{dt} = (v_a - R i_a - K_m [illegible])/L$

$\frac{di_b}{dt} = (v_b - R i_b - K_m [illegible])/L$

CAN BE VISUALLY INTERPRETED FROM $\frac{d\omega}{dt}$ [illegible] SHOWN ON SHEET 1

$$\frac{d\omega}{dt} = \left[-K_m i_1 \sin(N_r\theta) - K_m i_2 \sin(N_r\theta - \gamma) - K_m i_3 \sin(N_r\theta - 2\gamma) - B\omega\right]/J$$

$$i_1 = \frac{2}{3} i_{sa}$$

$-\sin(N_r\theta)$

$$+ \frac{1}{3} K_m i_{sa} \left(\sin(N_r\theta - \gamma) + \sin(N_r\theta - 2\gamma)\right)$$
$$- \frac{1}{3} K_m i_{sb} \left(\sin(N_r\theta - \gamma) - \sin(N_r\theta - 2\gamma)\right)$$

FROM PAGE 230 OF LEONHARD "CONTROL OF ELECTRICAL DRIVES"

$$i_{s1} = \frac{2}{3} i_{sa}$$
$$i_{s2} = -\frac{1}{3} i_{sa} + \frac{1}{\sqrt{3}} i_{sb}$$
$$i_{s3} = -\frac{1}{3} i_{sa} - \frac{1}{\sqrt{3}} i_{sb}$$

$+\cos(N_r\theta)$

$$\frac{d\omega}{dt} = \left[-K_m i_{sa} \sin(N_r\theta) + K_m i_{sb} \cos(N_r\theta) - B\omega\right]/J$$

CAN BE VISUALLY INTERPRETED FROM SINE WAVE CHARTS ON SHEET 1

3 PHASE / 2 PHASE MOTOR REPRESENTATION

8

$$\frac{di_a}{dt} = (v_a - Ri_a + 1.5\,K_m\,\omega \sin(N_r\theta))/L$$

$$\frac{di_b}{dt} = (v_b - Ri_b - 1.5\,K_m\,\omega \cos(N_r\theta))/L$$

$$\frac{d\omega}{dt} = (-K_m i_a \sin(N_r\theta) + K_m i_b \cos(N_r\theta) - B\omega)/J$$

$$\frac{d\theta}{dt} = \omega$$

A-B | D-Q TRANSFORMATION

$x^2 y^2 = \cancel{2x\,x^2 \ldots}\, y^2$

$$\frac{di_d}{dt} = \cos\theta \frac{di_a}{dt} - \cancel{\sin\theta\, i_a \frac{d\theta}{dt}}\ i_q$$
$$+ \sin\theta \frac{di_b}{dt} + \cancel{\cos\theta\, i_b \frac{d\theta}{dt}}$$

$$\frac{di_q}{dt} = -\sin\theta \frac{di_a}{dt} \cancel{- \cos\theta\, i_a \frac{d\theta}{dt}}\ -i_d$$
$$+ \cos\theta \frac{di_b}{dt} \cancel{- \sin\theta\, i_b \frac{d\theta}{dt}}$$

$$V_d = \cancel{\cos\theta R i_a} \overset{i_d}{} + \cos\theta \underset{A}{\overset{1.5}{K_m}} \omega \sin\theta \cancel{+ \cos\theta L \frac{di_a}{dt}} \quad \left(\frac{di_d}{dt} + i_q \frac{d\theta}{dt}\right)L$$
$$\cancel{+ \sin\theta R i_b} + \cancel{\sin\theta \underset{A}{\overset{1.5}{K_m}} \omega \cos\theta} + \sin\theta L \frac{di_b}{dt}$$

$$V_q = \cancel{-\sin\theta R i_a} \overset{i_q}{} + \sin\theta \underset{A}{\overset{1.5}{K_m}} \omega \sin\theta - \sin\theta L \frac{di_a}{dt} \quad 1.5 K_m \omega \quad \left(\frac{di_q}{dt} + i_d \frac{d\theta}{dt}\right)L$$
$$\cancel{+ \cos\theta R i_b} + \cancel{\cos\theta \underset{A}{\overset{1.5}{K_m}} \omega \cos\theta} + \cancel{\cos\theta L \frac{di_b}{dt}}$$

SEE PREVIOUS PAGE OF NOTES.